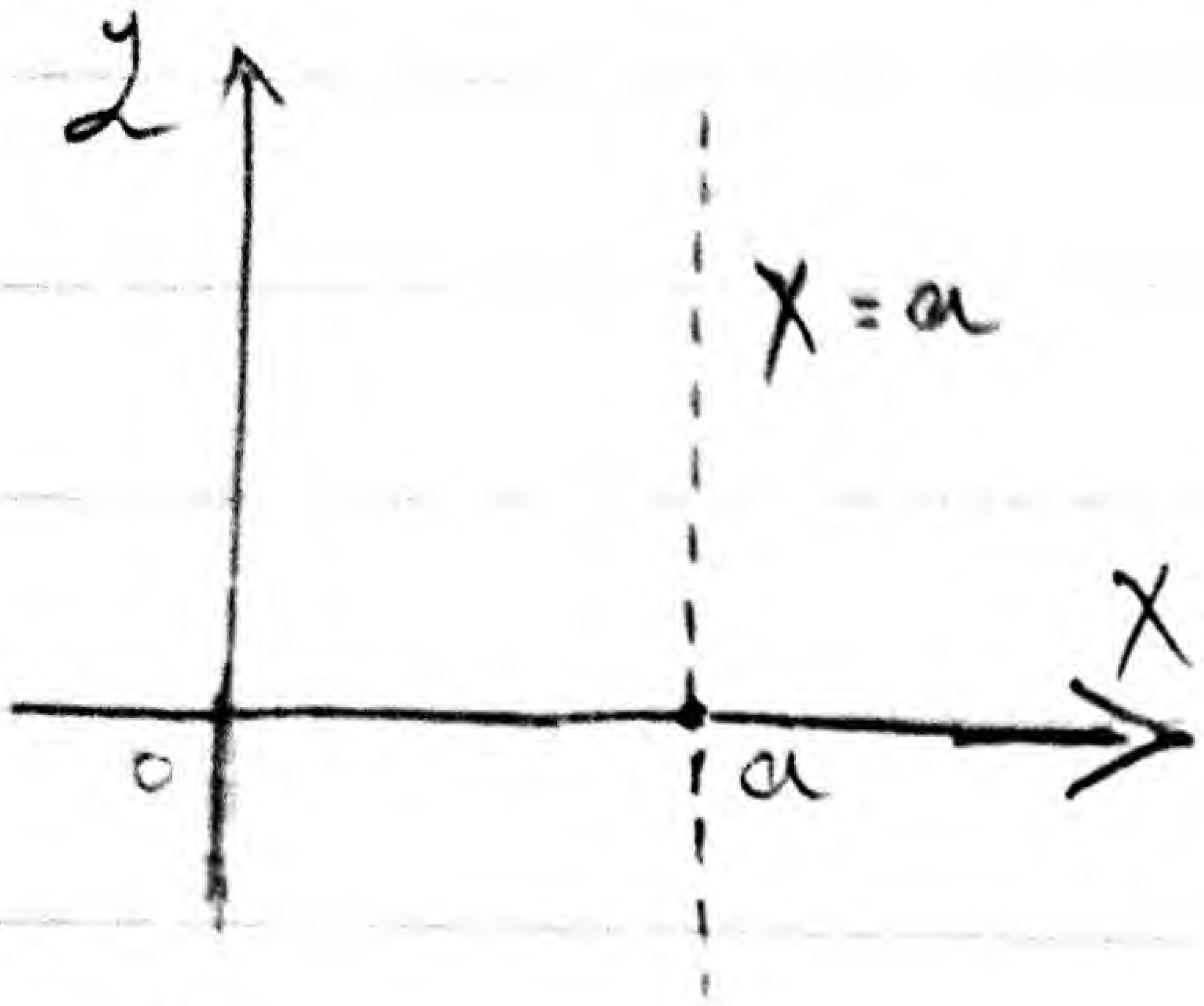
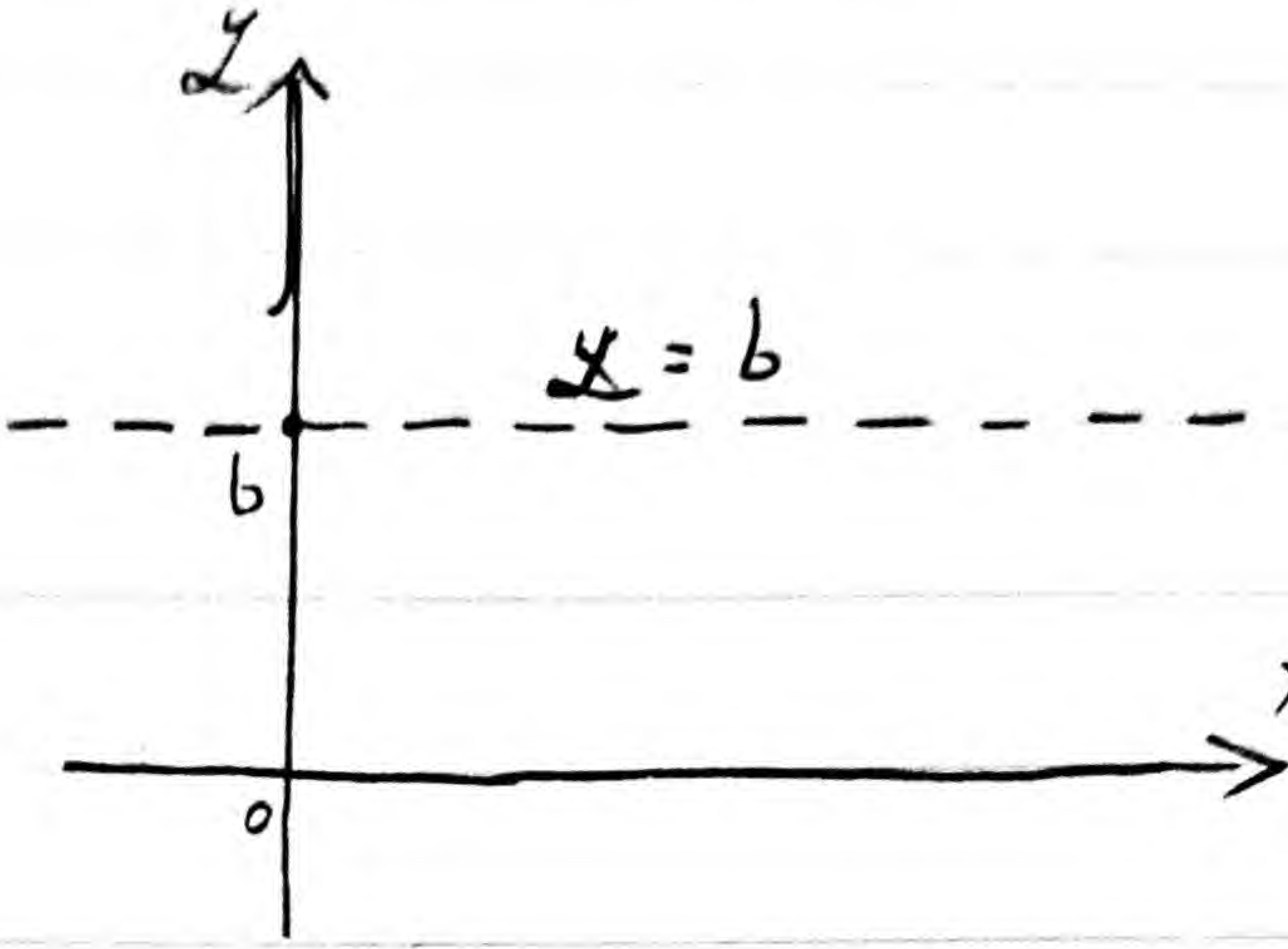
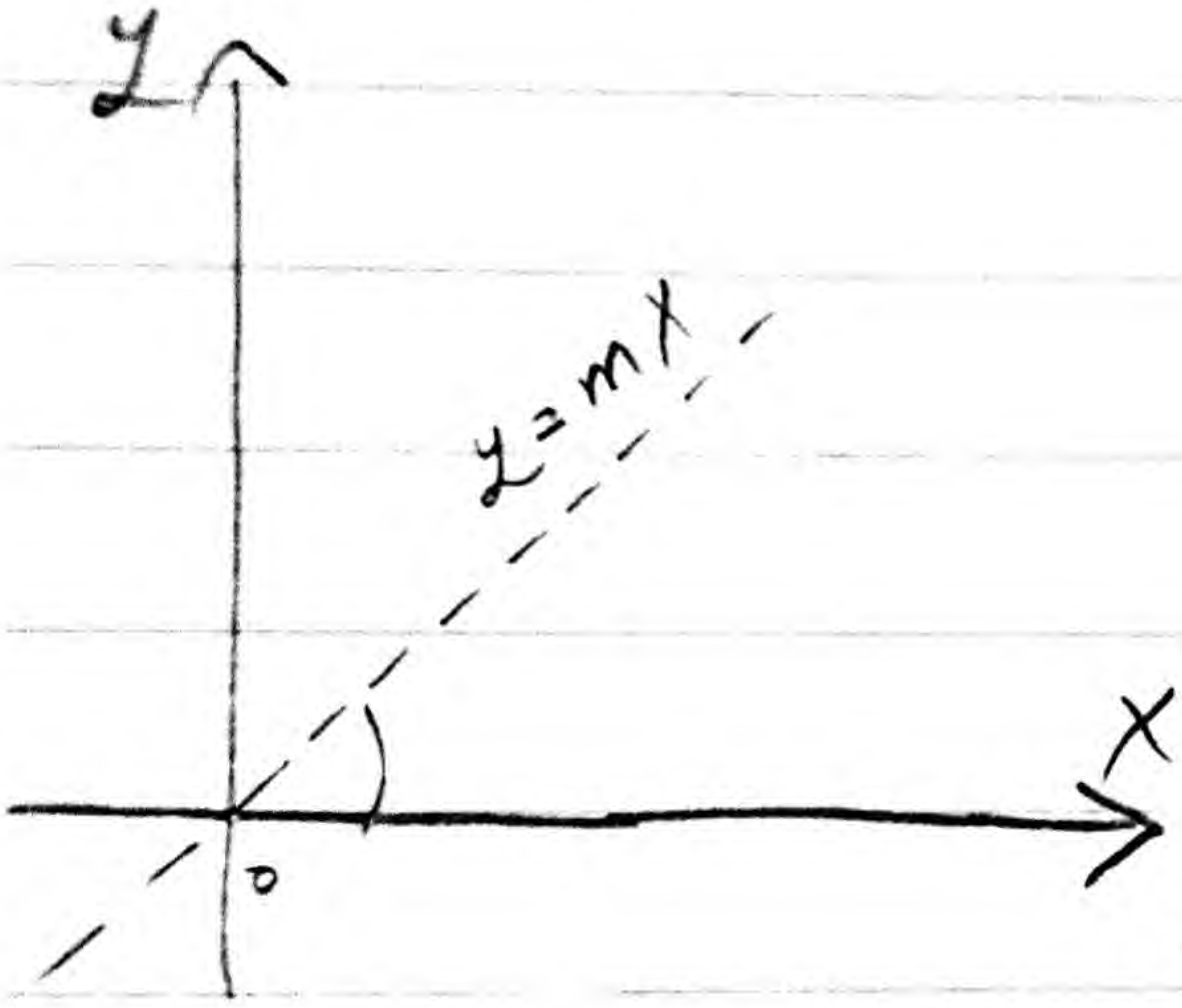
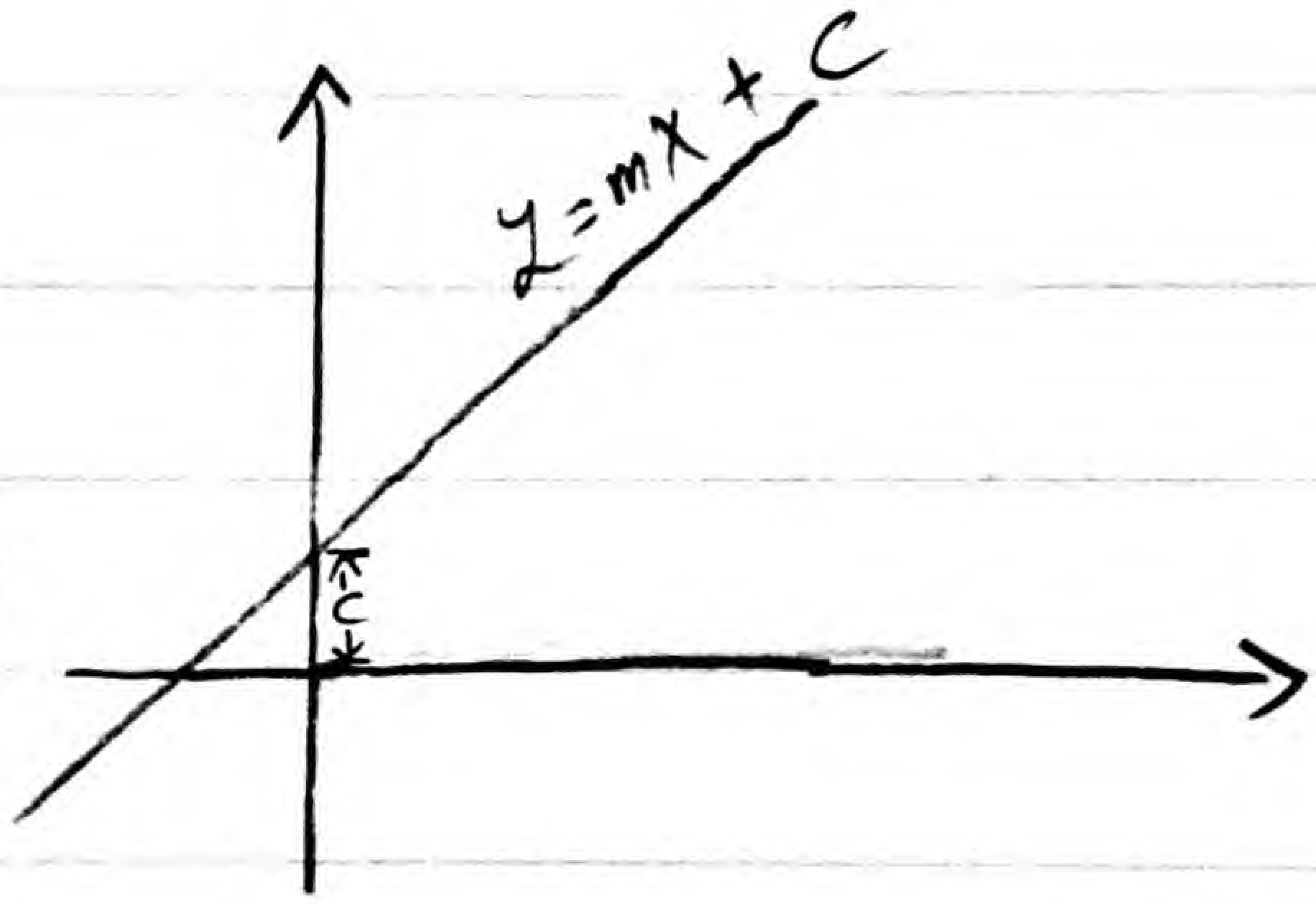
Subject. Date.

Two fold (Double) eqn of two stright Lines:-









$ax + by = 0$ $\qquad\qquad$ $ax + by + c = 0$

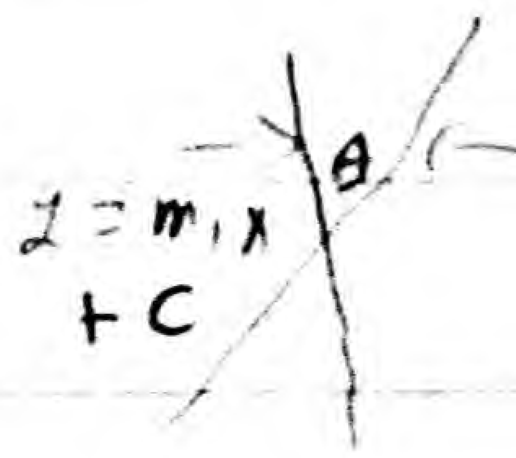
The two lines $L_1 : a_1x + b_1y + c_1 = 0$

& $L_2 : a_2x + b_2y + c_2 = 0$

are

a) // if $\frac{a_1}{a_2} = \frac{b_1}{b_2}$ $\qquad$ b) The Same if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1m_2}$$



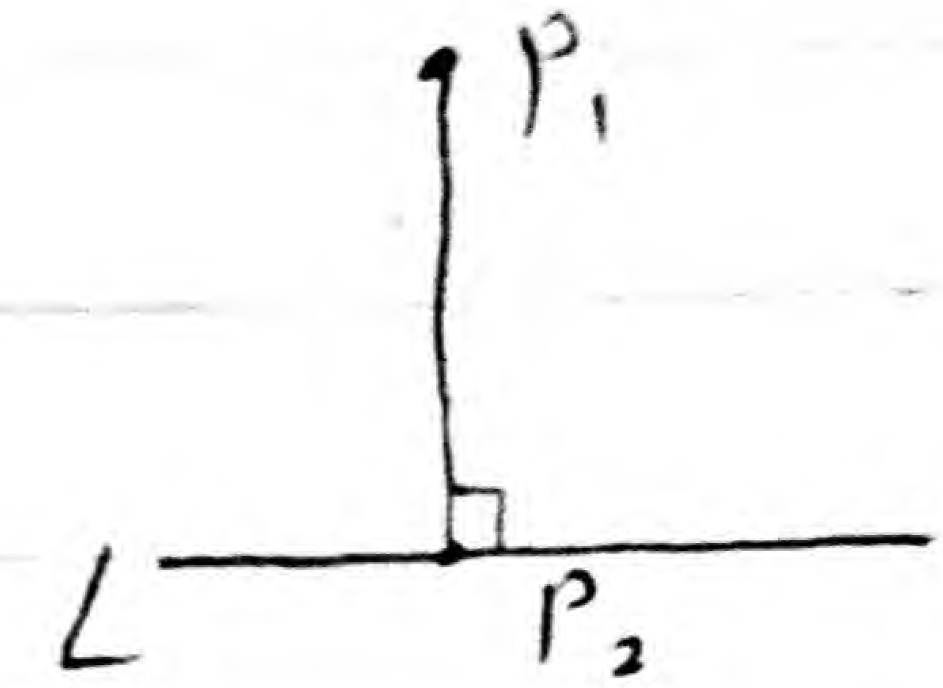
$y = m_2x + c_2$

* The distance bet. $P_1(x_1, y_1)$ & $P_2(x_2, y_2)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

* The distance from $P_1(x_1, y_1)$ to $L: ax + by + c = 0$ is

$$d = \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}$$



Ex: Find the line // L to $4x - 5y + 1 = 0$ passing thr (1, 2)

Sol

Any lines // L to (1) is given by $4x - 5y + K = 0$ (1, 2)

$4 - 10 + K = 0$

$K = 6$

$4x - 5y + 6 = 0$ #

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The homogeneous eqn of 2nd degree in X, y represents two lines passing thr O. $aX^2 + 2hXy + by^2 = 0 \to *$

بالقسمة على X^2

$$b\left(\frac{y}{x}\right)^2 + 2h\left(\frac{y}{x}\right) + a = 0$$

$$\frac{y}{X} = \frac{-2h \pm \sqrt{4h^2 - 4ab}}{2b}$$

$$\frac{y}{X} = \frac{\cancel{2}(-h \pm \sqrt{h^2 - ab})}{\cancel{2}b}$$

$$y = \left(\frac{-h \pm \sqrt{h^2 - ab}}{b}\right) X$$

$y = m_1 X$ & $y = m_2 X$

$$m_1 = \frac{-h + \sqrt{h^2 - ab}}{b}$$

$$m_2 = \frac{-h - \sqrt{h^2 - ab}}{b}$$

Provided $h^2 - ab \geq 0$

لكي تمثل خطين لازم المميز يكون
أكبر من أو يساوى صفر

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EX: Find the two lines represented by

$$y^2 - 7xy + 10x^2 = 0$$

$$(y - 2x)(y - 5x) = 0$$

$$y = 2x \quad \& \quad y = 5x$$

Note:- $m_1 + m_2 = \frac{-2h}{b}$

$$m_1 m_2 = \frac{a}{b}$$

The angle θ bet. the two lines ✱

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2} = \frac{\sqrt{(m_2 - m_1)^2}}{1 + m_1 m_2}$$

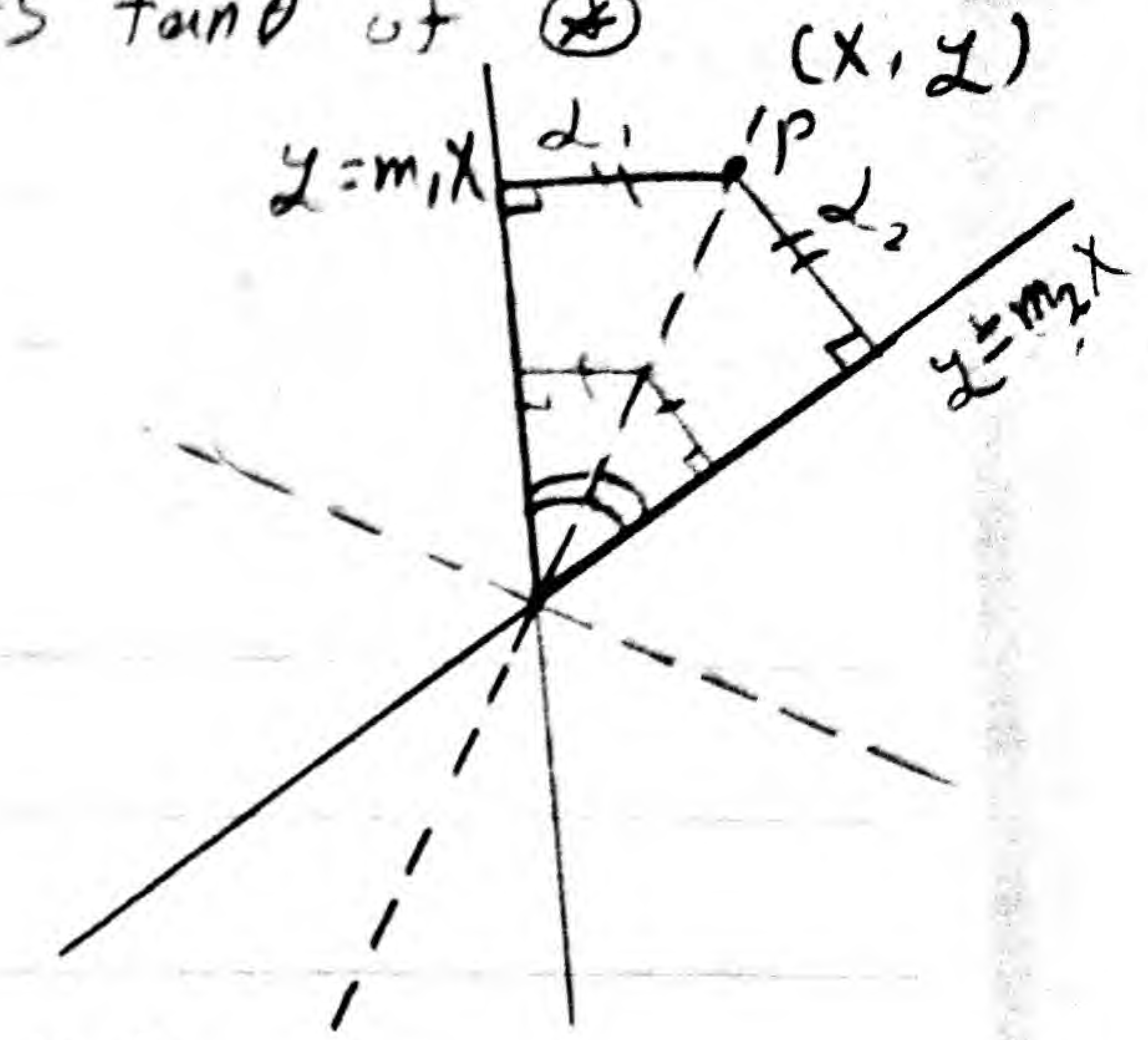
$$= \frac{\sqrt{(m_1 + m_2)^2 - 4m_1 m_2}}{1 + m_1 m_2}$$

$$\tan\theta = \frac{\sqrt{\frac{4h^2}{b^2} - \frac{4a}{b}}}{1 + \frac{a}{b}} = \frac{2\sqrt{h^2 - ab}}{a + b}$$

$\therefore a = 10, \; b = 1, \; h = \frac{7}{2}$

$$\tan\theta = \frac{2\sqrt{\frac{49}{4} - 10}}{11} = \frac{3}{11}$$

The double eqn of the bisectors tan θ of (*)



$d_1 = \pm d_2$ المنصفين

$$\left(\frac{y - m_1 x}{\sqrt{1 + m_1^2}}\right) = \pm \frac{y - m_2 x}{\sqrt{1 + m_2^2}}$$

$$\left(\frac{y - m_1 x}{\sqrt{1+m_1^2}} - \frac{y - m_2 x}{\sqrt{1+m_1^2}}\right)\left(\frac{y - m_1 x}{\sqrt{1+m_1^2}} + \frac{y - m_2 x}{\sqrt{1+m_2^2}}\right) = 0$$

$$\frac{(y - m_1 x)^2}{1 + m_1^2} - \frac{(y - m_2 x)^2}{1 + m_2^2} = 0$$

$$(1 + m_2^2)(y^2 + m_1^2 x^2 - 2 m_1 x y) - (1 + m_1^2)(y^2 - 2 m_2 x y + m_2^2 x^2) = 0$$

$$(m_1 + m_2)(x^2 - y^2) + 2(m_1 m_2 - 1) x y = 0$$

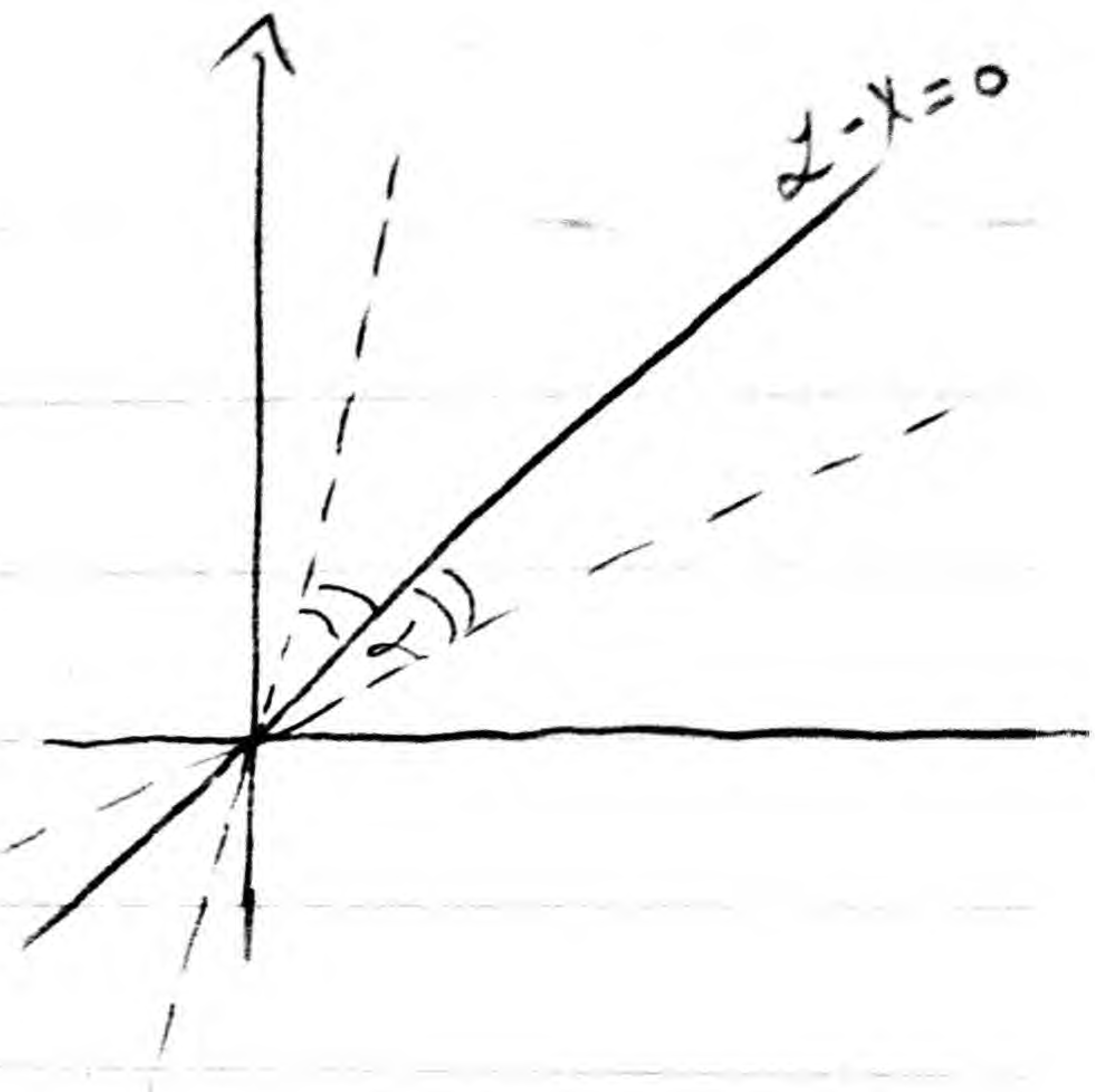
$$-\frac{2h}{b}(x^2 - y^2) + \cancel{2}\left(\frac{a}{b} - 1\right) x y = 0 \qquad (* b$$

$$h(x^2 - y^2) = (a - b) x y$$

$$\therefore \frac{x^2 - y^2}{a - b} = \frac{x y}{h}$$

X1- Prove that the two lines thro. and make an angle with the line $x - y = 0$ is given by

$x^2 \pm 2xy \sec 2\alpha + y^2 = 0$



$ax^2 + 2hxy + by^2 = 0$

$x - y = 0 \Rightarrow x + y = 0$

$x^2 - y^2 = 0$ ------ (1)

But the eqn of the bisectors

$x^2 - y^2 = \frac{a-b}{h} xy$ ----- (2)

(1) = (2)

$\frac{a-b}{h} = 0 \Rightarrow \boxed{a = b}$

$\tan 2\alpha = \frac{\cancel{2}\sqrt{h^2 - a^2}}{\cancel{2}a}$

$a^2 \tan^2 2\alpha = h^2 - a^2$

$h^2 = a^2(\tan^2 2\alpha + 1)$

$\boxed{h = \pm a \sec 2\alpha}$

$\cancel{a}x^2 \pm 2\cancel{a}\sec 2\alpha \cdot xy + \cancel{a}y^2 = 0$